Matrices

 $A = [a_{ij}]$

 $B = [b_{ij}]$

12

Definition: An m x n matrix is an arrangement of mn objects (not necessarily distinct) in m rows and n columns in the form

[a ₁₁	<i>a</i> ₁₂	 a_{1n}
A =	a ₂₁ :	a ₂₂	 <i>a</i> _{2<i>n</i>}
4	a _{ml}	a_{m2}	 a _{mn}

We say that the matrix is of order $m \times n$ (m by n). The objects $a_{11}, a_{12}, \ldots, a_{mn}$, are called the elements of the matrix. Each element of the matrix can be a real or a complex number or a function of one more variables or any other object. The element a; which is common to the *i* th row and the *j* th column is called its general element. The matrices are usually denoted by boldface uppercase letters , C, ... etc. When the order of the matrix is understood, we can simply write $A = [a_{ij}]$. If all elements of a matrix are real, it is called a real matrix, whereas if one or more elements of a are complex it is called a complex matrix.

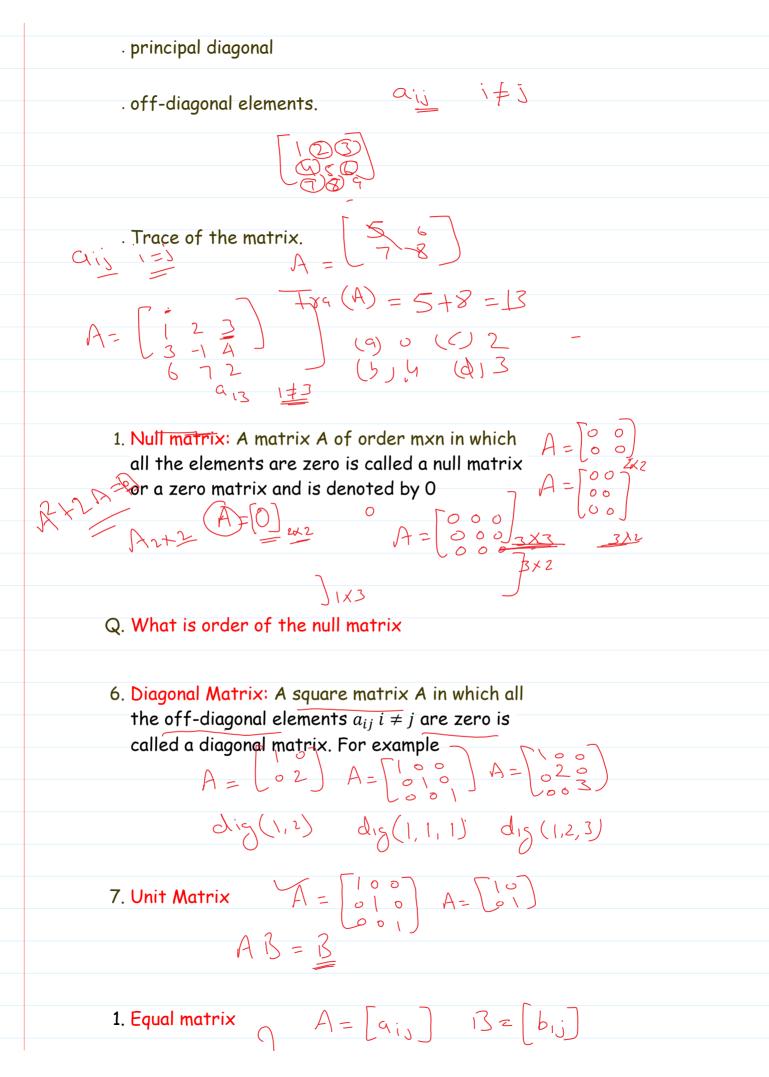
Types of Matrices

1. Row Vector: A matrix of order 1 x n that is, it has one row and n column is called row matrix or row vector. And it can be written as

 $[a_{11}, a_{12}, \dots, a_{1n}]$ in which a_{1j} is the j th element.

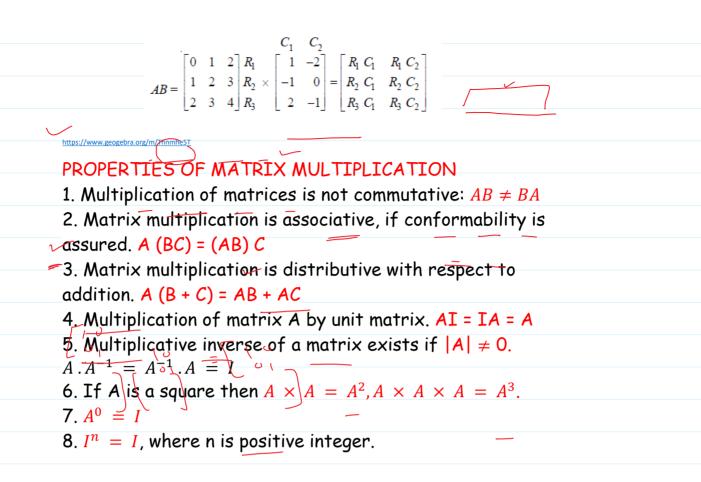
F1,2,3

•. What is the order of row vector ? 1. Column vector: A matrix of order mx1, that is, it has m row and one column is called column vector or column matrix of order m and is written as 🚺 •. What is the order of column vector ? 3. Rectangular matrix: A matrix A of order m x n, $m \neq m$ is called a rectangular matrix. $\frac{A}{12} = \begin{bmatrix} a_{ij} \end{bmatrix} m \times n \begin{bmatrix} 2 & y & b \\ 3 & 2 & b \end{bmatrix} \times 3$ $\frac{2}{12} = n \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = 2 + 3$ $\frac{2}{12} = n \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ 4. Square matrices: A matrix A of order m x n in which m = n, that is number of rows is equal to the number of columns is called a square matrix of order n. 21 $2x_2$ 78 $93x_3$ $[5]_{1x_1}$. diagonal elements aij i=j . principal diagonal



1. Equal matrix
$$A = \begin{bmatrix} a_{1,3} \\ \vdots \end{bmatrix}$$
 $B = \begin{bmatrix} b_{1,j} \\ \vdots \end{bmatrix}$
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 $\int A = \left[\overline{A_{ij}} \right]$ kA = a ka kbkA = [kai]KC KO https://www.geogebra.org/m/jaJwgaar A= [a, j]mxh = [bij]man A+15 = [a; j+bij] (ii) Addition/subtraction of two matrices, $A = \begin{bmatrix} 4 & 2 & 5 \\ 1 & 3 & -6 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ $A + B = \begin{bmatrix} 4+1 & 2+0 & 5+2 \\ 1+3 & 3+1 & -6+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 7 \\ 4 & 4 & -2 \end{bmatrix}$ Note: Only matrices of the same order can be added or subtracted. (i) Commutative Law: A + B = B + A. (ii) Associative law: A + (B + C) = (A + B) + C. Given 3 $\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} x + y & -1 & 2w + 3 \end{bmatrix}$ Find x, y, z and w. a. 2,4,1,3 b. 2,1,3,4 c. 4,2,3,4 d. 1,3,2,4



Some special Matrices

Transpose of the matrix: If in a given matrix A, we interchange the rows and the corresponding columns, the new matrix obtained is called the transpose of the matrix A and $\frac{3}{4}$ 5 8 is denoted by A' or A^T $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

Then A' or $A^T = ?$
Symmetric Matrix: A square matrix will be called symmetric,
if for all values of i and jo
 $a_{ij} = a_{ji}$ i.e., $A = A^T + A^T$

 $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

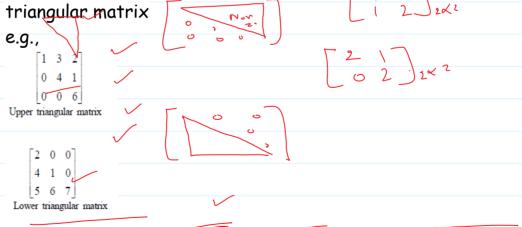
Skew symmetric matrix: A square matrix is called skew symmetric matrix, if

(1) $a_{ij} = -a_{ji}$ for all values of i and j, or $A^T = -A$

(2) All diagonal elements are zero,

$\begin{bmatrix} 0 & -h & -g \end{bmatrix}$			
h 0 -f			
-			
$\lfloor g f 0 \rfloor$			

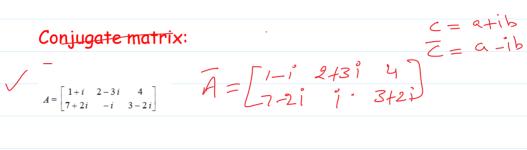
Triangular matrix: (Echelon form) A square matrix, all of whose elements below the leading diagonal are zero, is called an upper triangular matrix. A square matrix, all of whose elements above the leading diagonal are zero, is called a lower

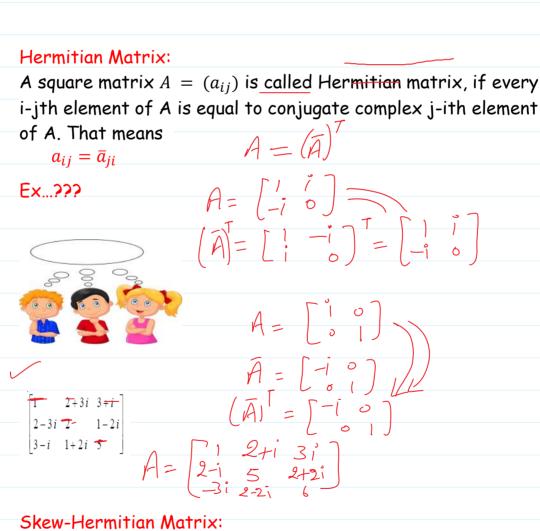


Orthogonal Matrix: A square matrix A is called an orthogonal matrix if the product of the matrix A and the transpose matrix A' is an identity matrix e.g. $A \cdot A^T = I$ EX... ???? $F \times I$ $A = \begin{bmatrix} I & O \\ O \end{bmatrix}$



Note: if | A | = 1, matrix A is proper.



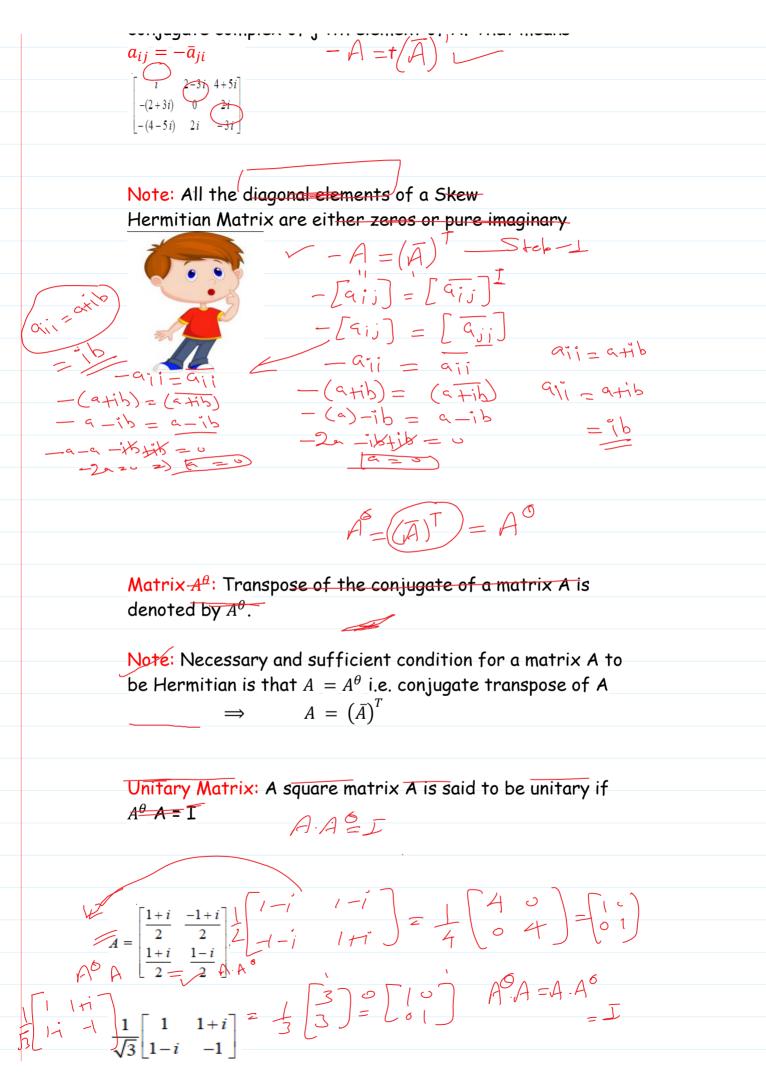


Skew-Hermitian Matrix:

A square matrix $A = (a_{ii})$ will be called a Skew Hermitian matrix if every i-jth element of A is equal to negative conjugate complex of j-ith element of A. That means -A = t/A)'L

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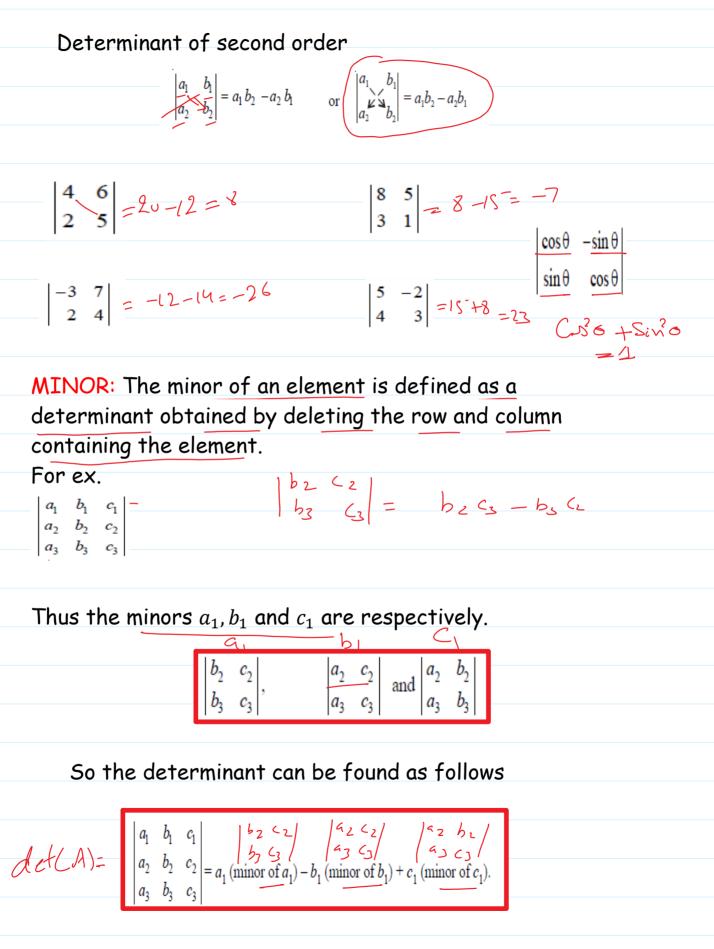
 $a_{ii} = -\bar{a}_{ii}$

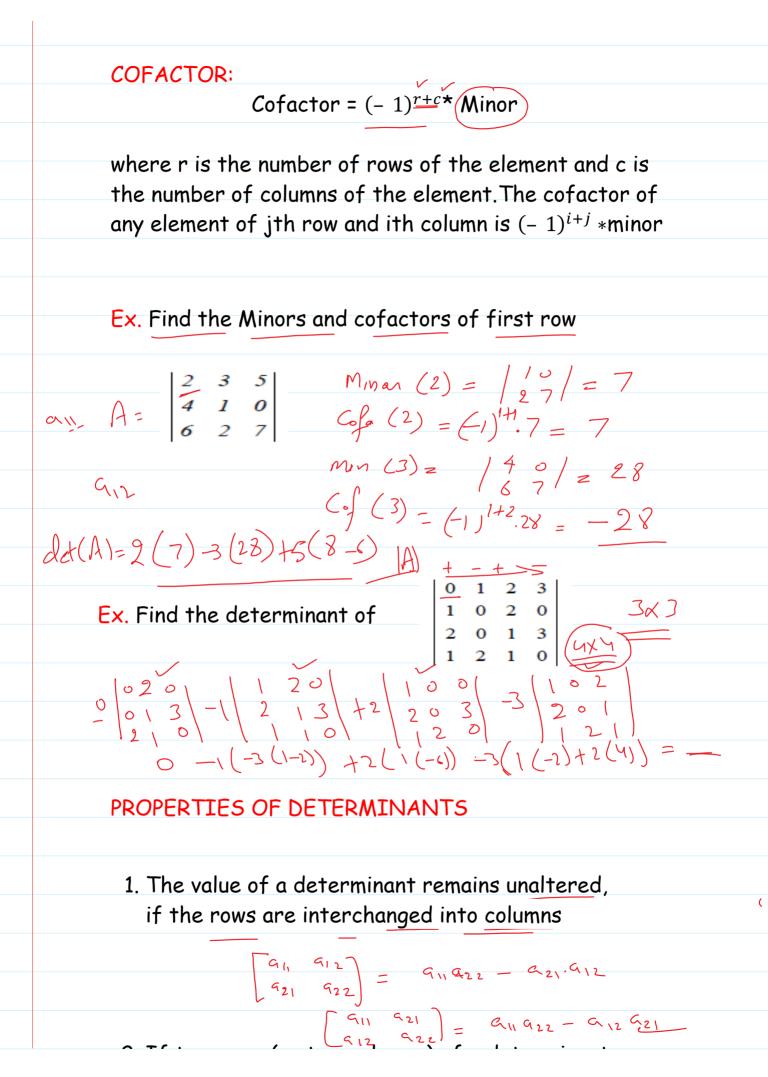


Identify
$$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 3 + 2 - 4 & 1 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 3 + 4 & 1 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

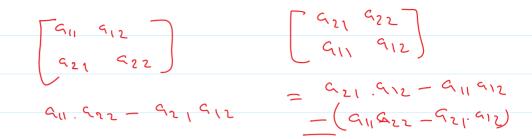
So we can say: Unit matrix is involuntary. $-A = \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix} \begin{bmatrix} \mathcal{L} & -\mathcal{L} \\ \mathcal{L} & -\mathcal{L} \end{bmatrix} = \begin{bmatrix} \mathcal{L} & \mathcal{O} \\ \mathcal{O} & \mathcal{L} \end{bmatrix}$ $A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} \xrightarrow{3}$ Singular Matrix: If the determinant of the matrix is zero, then the matrix is known as singular matrix IAI on det(A) = ~ Singular IAI to Non Sim

Determinant





2. If two rows (or two columns) of a determinant are interchanged, the sign of the value of the determinant changes.



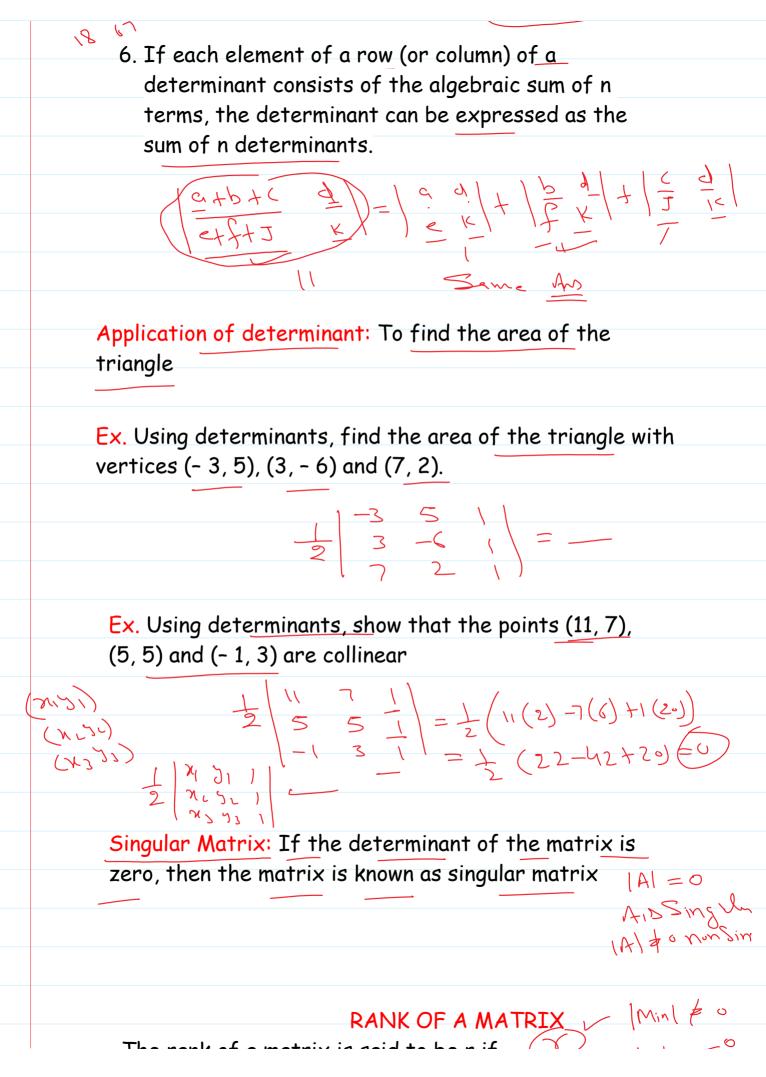
|9 9| = 0 |9 K9| = Kbq-kbe = -

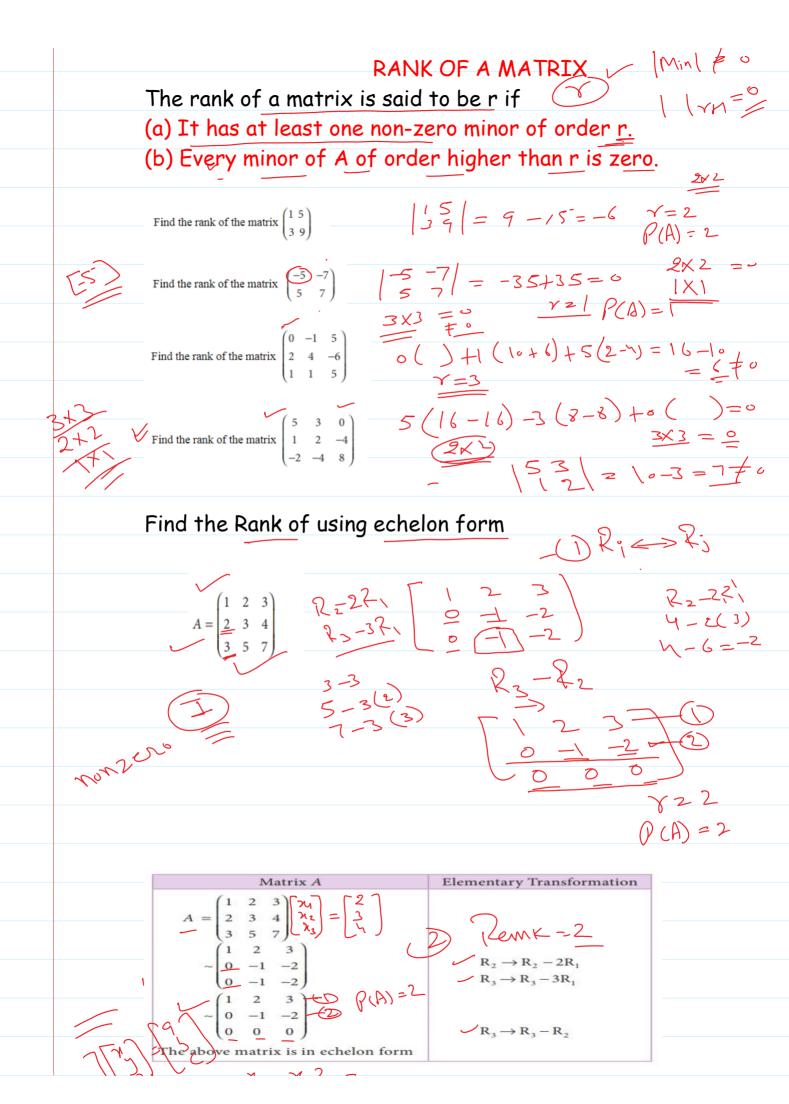
 $\left(\begin{array}{c} Ka & Kb \\ C & d \end{array}\right) = Kad - Kb(\\ - (K(ad-b(l)))$

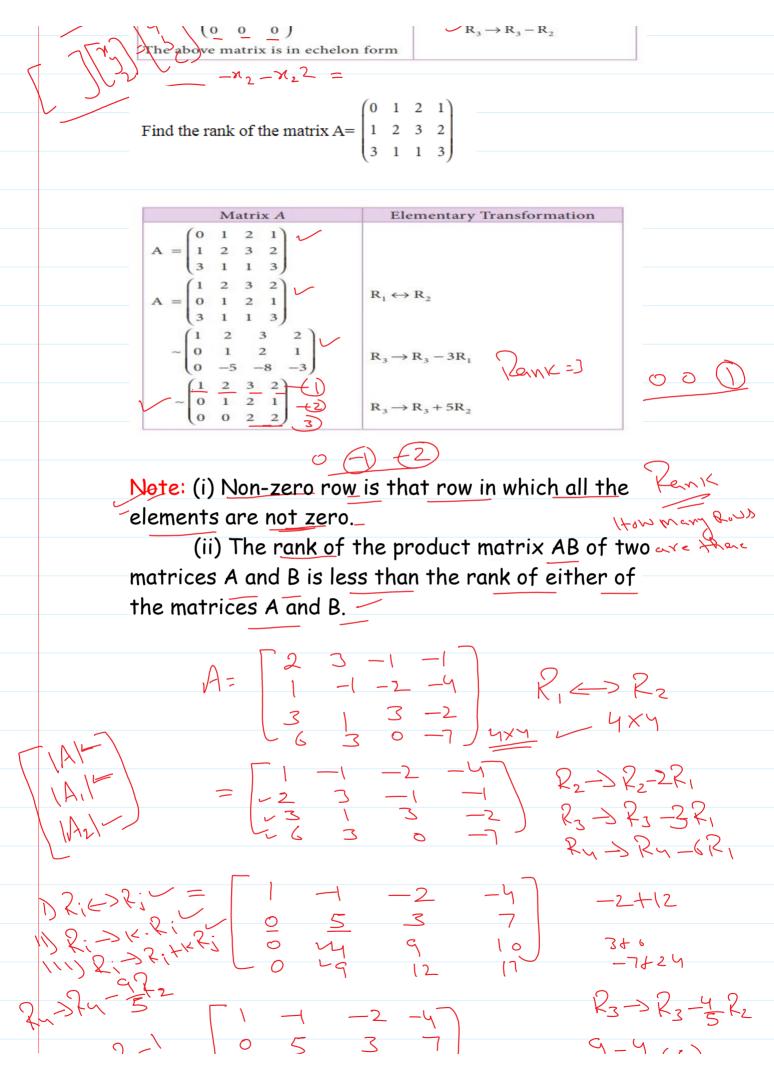
3. If two rows (or columns) of a determinant are identical, the value of the determinant is zero.

- 5. The value of the determinant remains unaltered if to the elements of one row (or column) be added any constant multiple of the corresponding elements of any other row (or column) respectively.

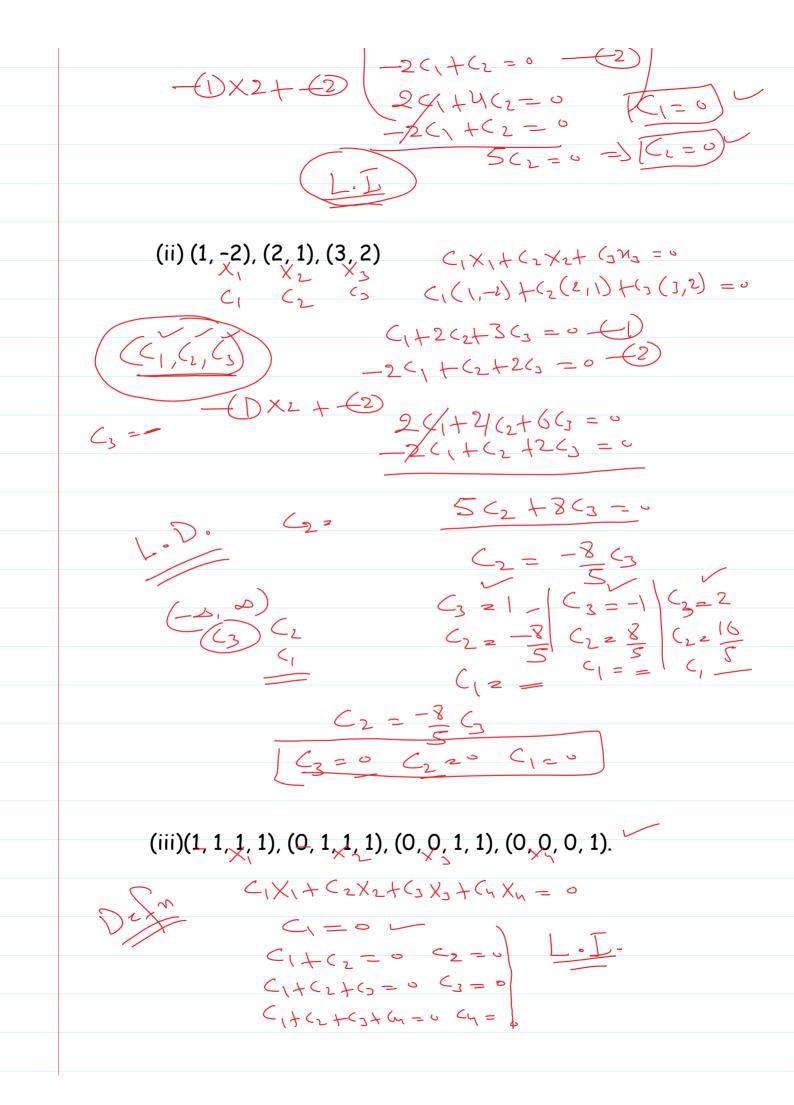
 $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a + kb & b \\ c + k & d \end{bmatrix}$ $\begin{bmatrix} y & 2 \\ -y \end{bmatrix} = \begin{bmatrix} a & -b \\ c & -kb \\ -b \\ -kb & -$



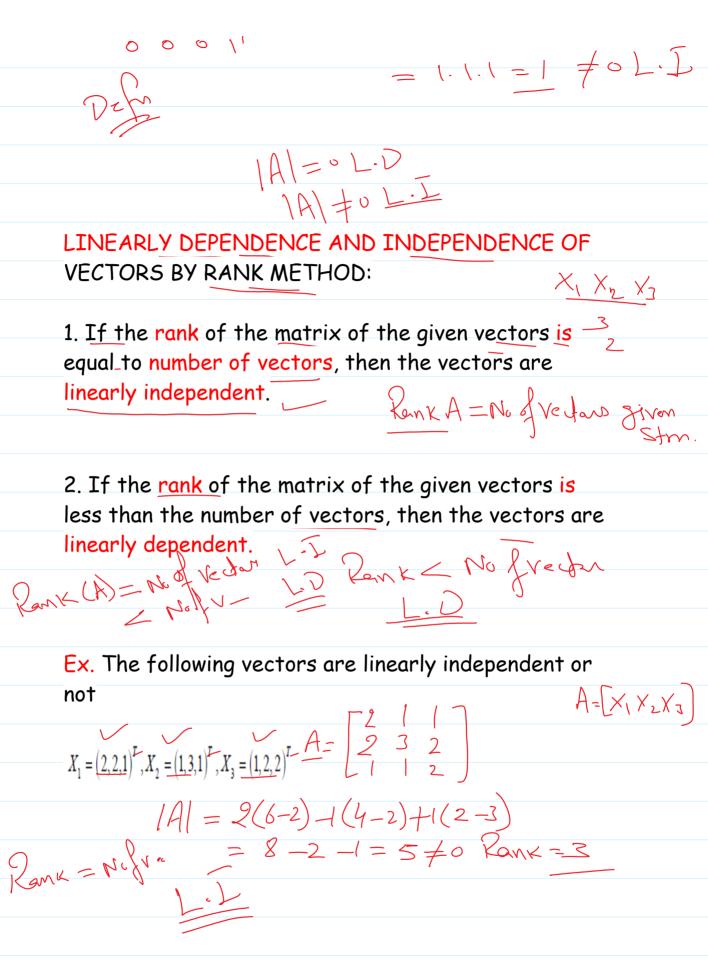




K3-3K3-5K2 0 5 3 $\gamma - \frac{\gamma}{5}(s)$ $\frac{4r-12}{r} = \frac{33}{5}$ 10-47 50-28 Linearly dependence and independence of vectors: X1 = [a11 412 413] X1 = [a11 a12] YOU MAAR IXM YOU MAIN Vectors (matrices) X_1, X_2, \dots, X_n are said to be (1) all the vectors (row or column matrices) are of the $\times [1,2,3] (1,2)$ same order. (2) n scalars $C_1, C_2, \ldots C_n$ (not all zero) exist such that $\underline{C_1}X_1 + \underline{C_2}X_2 + \dots + \underline{C_n}X_n = 0$ $C_{i=\zeta_{2}=\zeta_{3}=\ldots}=(n=0 \quad L\cdot I^{\perp})$ but if ableast one Cito L.D. L.D LX C Otherwise they are linearly independent. Find whether or not the following set of vectors are linearly dependent or independent: (i) (1, -2), (2, 1) $\frac{C_{1}X_{1} + C_{2}X_{2} = 0}{C_{1}(1,-2) + C_{2}(2,1) = 0}$ $C_{1}+2(2=0)$ --2(1+(2=0))



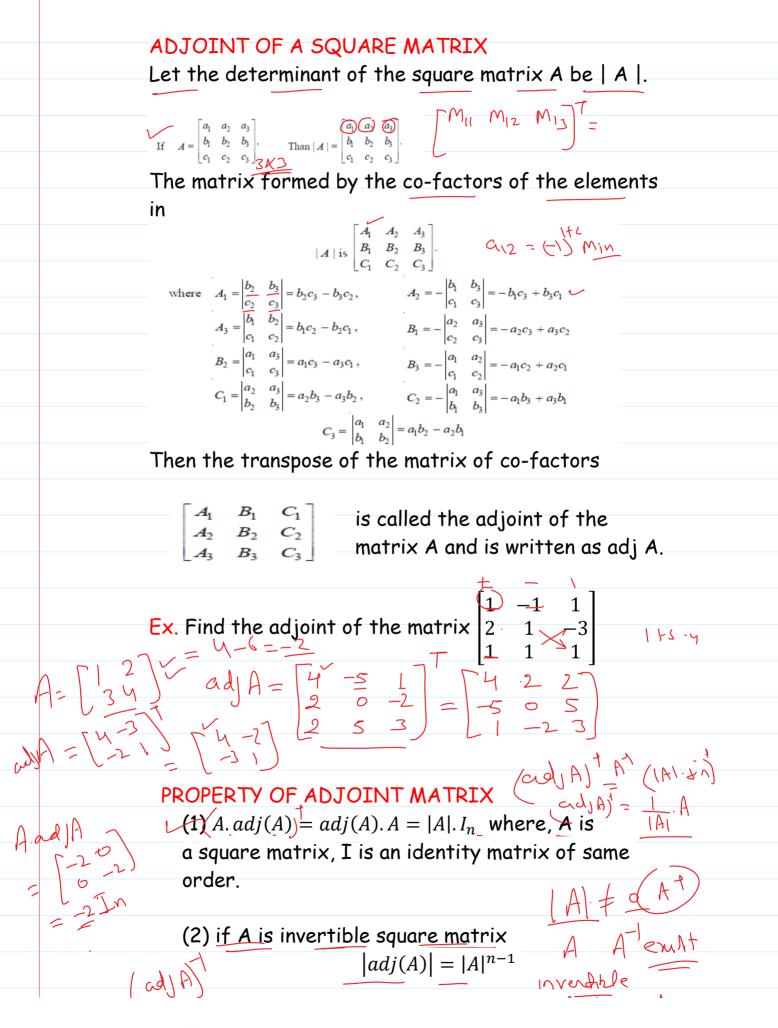
LINEARLY DEPENDENCE AND INDEPENDENCE OF VECTORS BY DETERMINANT: |A|=0 If the determinant of the these $X_1, X_2, X_3, \dots, X_n$ is zero then they are dependent otherwise independent. 1A1=1A1 **Ex.** (i) (1, -2), (2, 1) $|A| = 0 L \cdot D$ $\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 1 + 4 = \frac{5}{2} \neq 0 \quad L \cdot Z \qquad \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix}$ (3,3,2) (1,2,1) (2,1,1)(ii)(1,2,1),(2,1,1)(3,3,2) (ii)(1,2,1),(2,1,1)(3,3,2) (1,2,1),(2= 1(2-3)-2(4-3)+1(6-3) $= -1 - 2(1) + 3 = 0 L \cdot D$ IAI zlat 3 (0+2)-1(+2)+2(-2) (-2-420 L.D , 1, 1, 1), (0, 1, 1, 1), (0, 0, 1, 1), (0, 0, 0, 1).



Ex. Show using a matrix that the set of vectors X = [1, 2, -3, 4], V = [3, -1, 2, 1], Z = [1, -5, 8, -7] islinearly dependent.

linearly dependent. y (2) 2 -18 2 -3 4 -5 8 -7 (A) = |13 JXY A 6×2 $\frac{Min(3,4)}{min(m,m)}$ G) (X 3t) 13)5× (c) \mathcal{Y} (λ) [(x, y, 2], 2n 312) 2 MK ZLX _____ 210 1 C1×1+C2×2+- $-C_{n}X_{n} \rightarrow$ 1 (5 Cr=o ×i L-I) at lantone (i to L.D

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$$|adj(A)| = |A|^{n-1}$$

$$|adj(A)| = |A|^{n-1}$$

$$|ATF, A^{T}_{0,nM}$$

$$(3) if A is invertible square matrix
$$|ATF, A^{T}_{0,nM}$$

$$adj(adj(A)) = |A|^{n-2} A$$

$$(A)^{n-1} |P|^{-1}$$

$$|A|^{n-1} |P|^{-1} |P|^{-1} |P|^{-1}$$

$$|A|^{n-1} |P|^{-1} |P|^{-1}$$

$$(A)^{n-1} |P|^{-1} |P|^{-1} |P|^{-1}$$

$$(A)^{n-1} |P|^{-1} |P|^{-1} |P|^{-1}$$

$$(A)^{n-1} |P|^{-1} |P|^{-1} |P|^{-1} |P|^{-1}$$

$$(A)^{n-1} |P|^{-1} |P|^{-1} |P|^{-1} |P|^{-1} |P|^{-1}$$

$$(A)^{n-1} |P|^{-1} |P|^{$$$$

A1(=1). 1. Inverse of the matrix is unique AB' = NA = L(AB) -2. $(AB)^{-1} = B^{-1}A^{-1}$ 3. If A is an inverti is also invertible 3. If A is an invertible square matrix; Then $(A)^T$ is also invertible and $(A^T)^{-1} = (A^{-1})^{T}$ $|A| = |A^{\mathsf{T}}|$ A 12 MV evalue 4. The inverse of an invertible symmetric matrix is a symmetric matrix. $\begin{bmatrix} f & b & h \\ S & h & c \end{bmatrix} = \begin{bmatrix} c & f & d \\ A & b & h \\ S & h & c \end{bmatrix} = \begin{bmatrix} c & f & d \\ A & b & h \\ C & b & c \end{bmatrix}$ 5. $|A^{-1}| = |A|^{-1}$ i.e. $|A^{-1}| = \frac{1}{|A|}$ 1A1 = (A1 A= [-03] IAI= 6 $A^{T} = \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \right\} \quad |A^{T}| = 1, \dots, [6]^{T} = 1$ $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad |A| = 8$ Solution of $n \times n$ linear system of equation Consider the system of n equations in n unknowns 2n57 = 5 $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \quad \bigcup \quad \frac{3n+ij}{4n+ij} = 8$ $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \quad z$ $a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n.$ 3 In matrix form we can write this system as Ax = b $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{ln} \\ a_{21} & a_{22} & \dots & a_{2n} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

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$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{ln} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

 $\frac{1}{4n-b+0}$

x=0 7=0

-[x=0,3=0,2=0) non zoro Sel

Note:

- 1. A is the coefficient matrix, b the right hand side, and x is the solution vector.
- 2. If b not equal to zero system is called nonhomogeneous.
- 3. If b is zero its call homogeneous.

MJ=

4. The system of equations is called consistent if it has at least one solution.

otherwise the system is inconsistent.

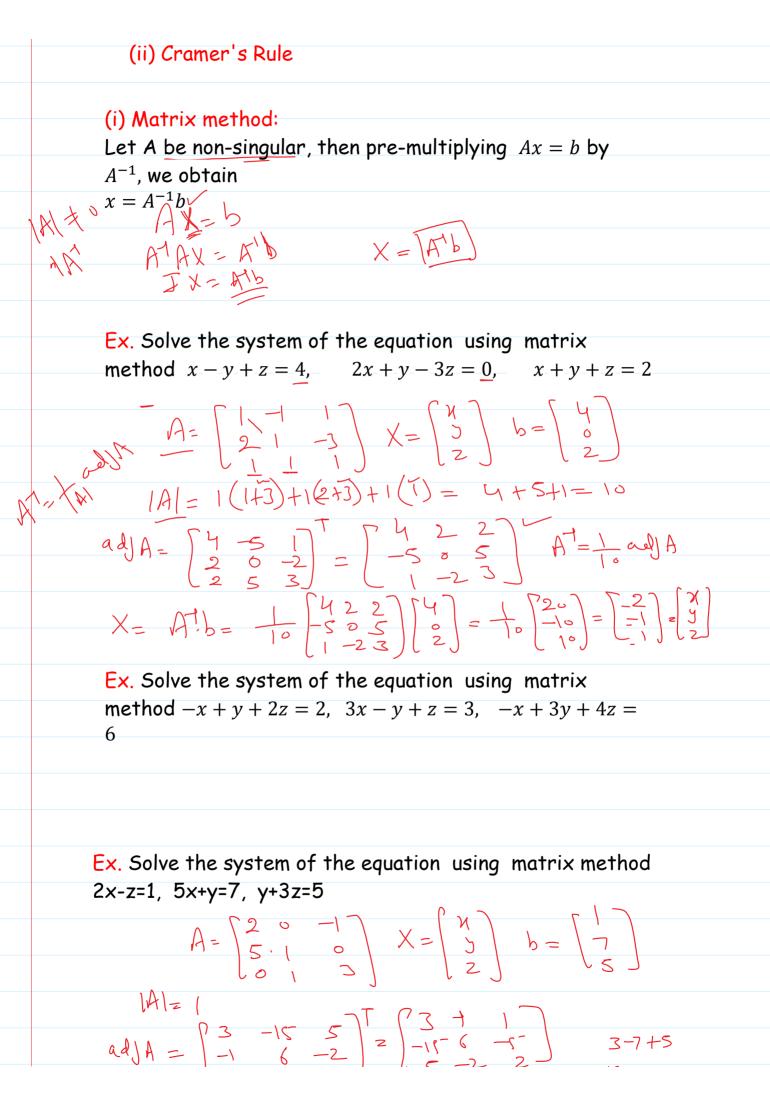
Homogeneous system of equations: Consider the homogeneous system of equations Ax = 0Trivial solution x = 0 is always a solution of this system.

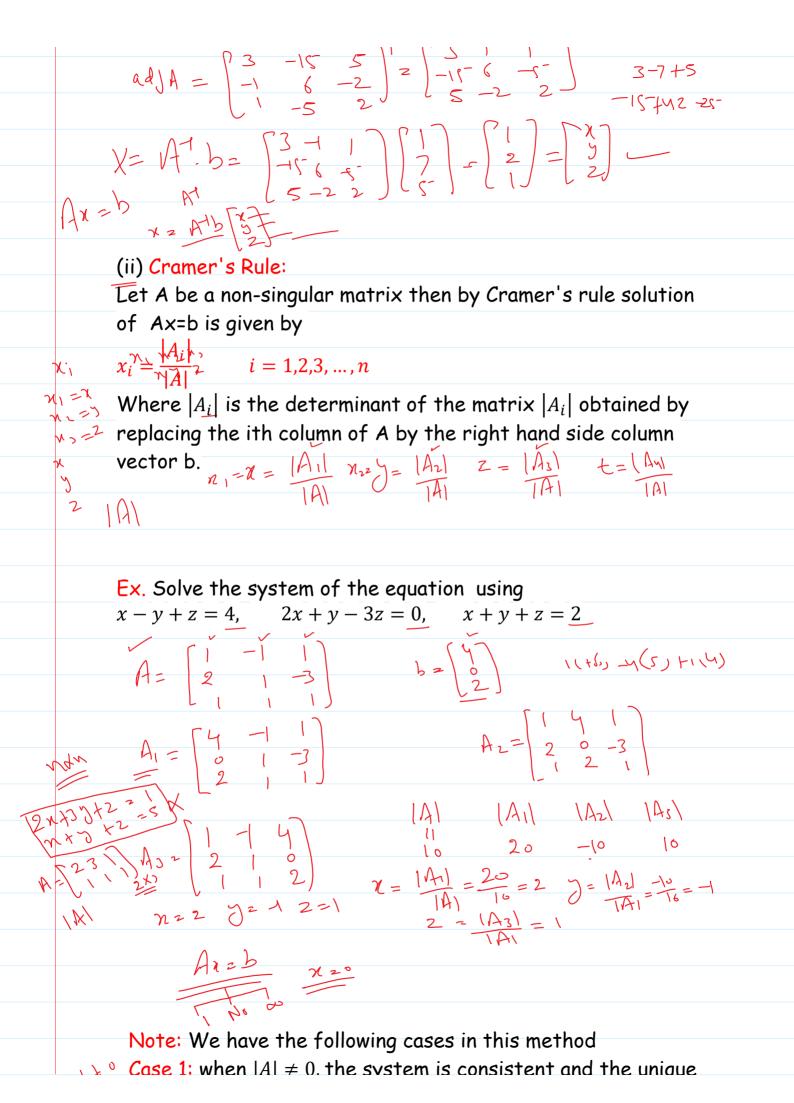
2nty =-3nty --If A is non singular, then $x = A^{-1}0 = 0$ is the solution.

Thus Ax = 0 is the always consistent.

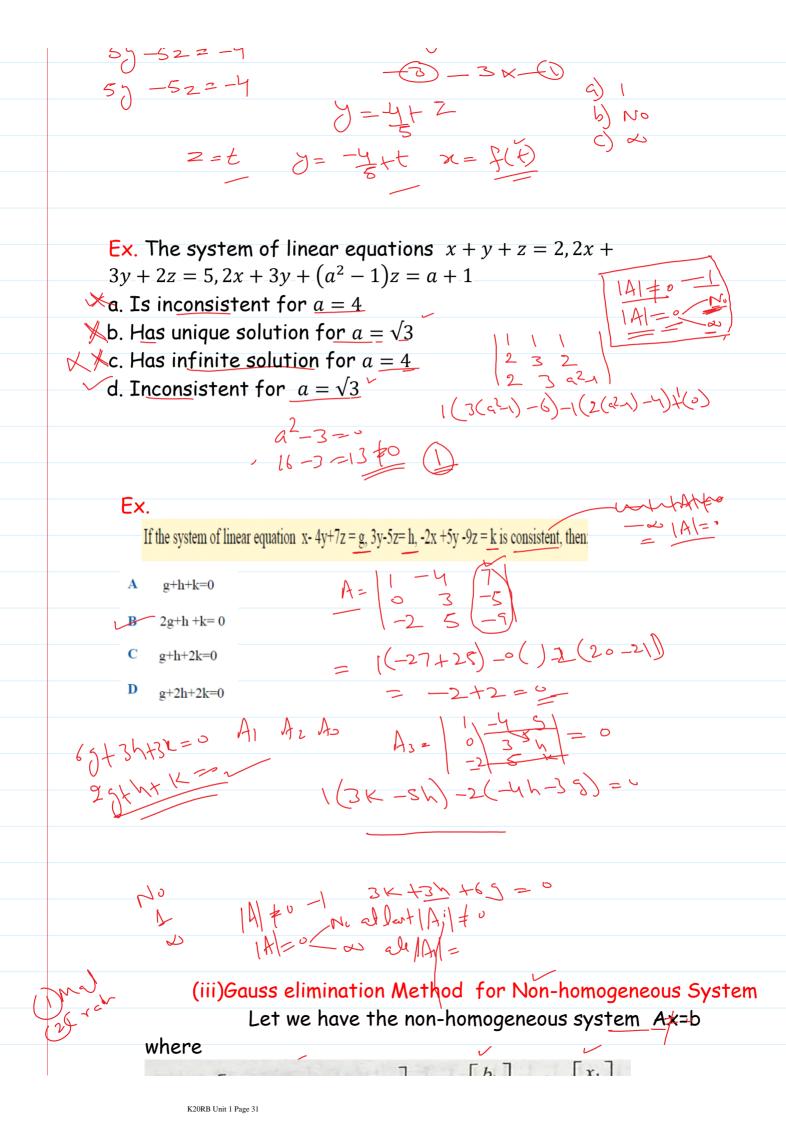
We conclude that non-trivial solution for Ax = 0 exist if and only if A is singular, in this case this system has infinite solutions. Ax = 0 Axsolutions.

2000 /A/=1070 x1y12=" Ex. Solve the system of the equation using $\underline{x} - \underline{y} + \underline{z} = 0$, 2x + y - 3z = 0, x + y + z = 0 $\chi = \chi^{2/2}$ $\begin{array}{c} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{array}{c} \chi \\ -2 \end{bmatrix} \quad \begin{array}{$ Ex. If the system of the equations x - ky - z = 0, kx - y - z = 0z = 0, x + y - z = 0 has non-zero solution then values of K |A| = 0 |A| = 0 |K - 1 - 1| = 0 $|K^{2} = 1$ $1(1 + 0 + K(-K + 1) - 1(K + 1)) |K^{2} = 1$ $2 - K^{2} + K - 1 |K^{2} = 1$ $- K^{2} + 1 = 0$ 1 of theare a. -1,2 b. 0,1 c. 1,1 d. -1,1 Ex. If the system of the equations kx + y + z = 0, -x + y + z = 0ky + z = 0, -x - y + kz = 0 has non-zero solution then value of K is |K|| = 0La. 0 b. 1 K(K2+1)-1(-K+1)H(1+1)=0 K(K2+1)+K/+K/K / K=0 K(K2+1+2)=0 / K=±JJi c. -1 d 2 Solution of Non-homogeneous system of equations An=b The non-homogeneous system of equations $Ax = \overline{b}$ can be solved by the following methods (i) Matrix method





Note: We have the following cases in this method Case 1: when $|A| \neq 0$, the system is consistent and the unique solution is obtained by using the above method. ALLAN-Case 2: When |A| = 0, and one or more of $|A_i|$, i = 1, 2, 3, ..., nare not zero then the system of the equations has no solution that is the system is inconsistent. **Case 3:** When |A| = 0, and all $|A_i| = 0$, i = 1, 2, 3, ..., n, then the system of equations is consistent and has infinite number of solutions . The system of equations has at least a one-parameter family of solutions. Ex. Solve the system of the equation using 4x + 9y + 3z = 6, 2x + 3y + z = 2, 2x + 6y + 2z = 7 $\left[\frac{1}{2}, \frac{3}{2}\right] = \left[A\right] = \left[\frac{4}{2}, \frac{9}{3}, \frac{3}{2}\right]$ $b = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ $0 = |A_1| = \begin{bmatrix} 6 & 9 & 3 \\ 2 & 3 & 1 \\ 7 & (2) \end{bmatrix} \quad 0 \neq \begin{bmatrix} A_1 \\ 2 & 2 & 1 \\ 4 & 2 & 7 \\ 2 & 7 & 2 \end{bmatrix}$ Ex. Solve the system of the equation using x - y + 3z = 3, 2x + 3y + z = 2, 3x + 2y + 4z = 5 $|A| = \begin{vmatrix} 1 - 1 & 3 \\ 2 & 3 & 1 \end{vmatrix} = 0 \qquad (A_1) = \begin{vmatrix} 3 & -1 & 3 \\ 2 & 3 & 1 \\ 5 & 7 & 4 \end{vmatrix} = 0$ $|A_{1}| = \begin{vmatrix} 1 & 3 & 3 \\ 2 & 2 & 1 \\ 3 & 5 & 4 \end{vmatrix} = 0 \qquad |A_{2}| = 2 & 3 & 2 \\ 3 & 2 & 5 \end{vmatrix} = 0$ (x - j + 32 = j - (j))(2x + 3y + 2 = 2 - (2))(3x + 2j + 42 = 5 - (3)) $-3 - 3 \times -0$ a) 1

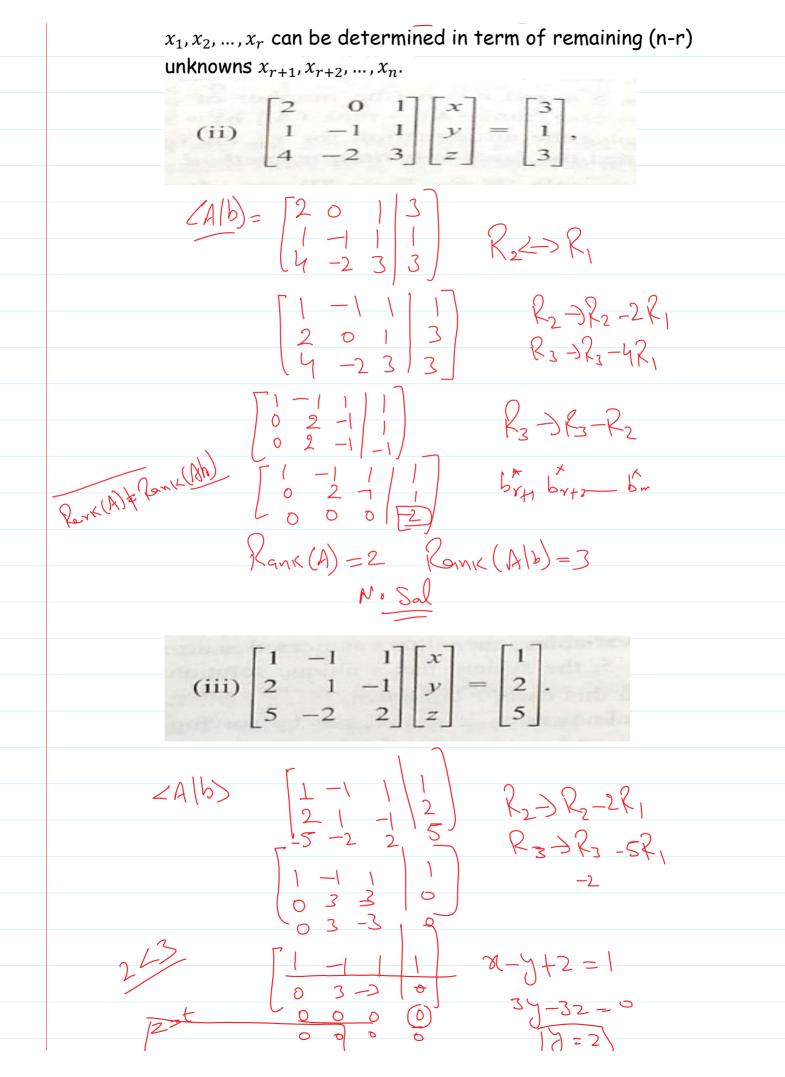


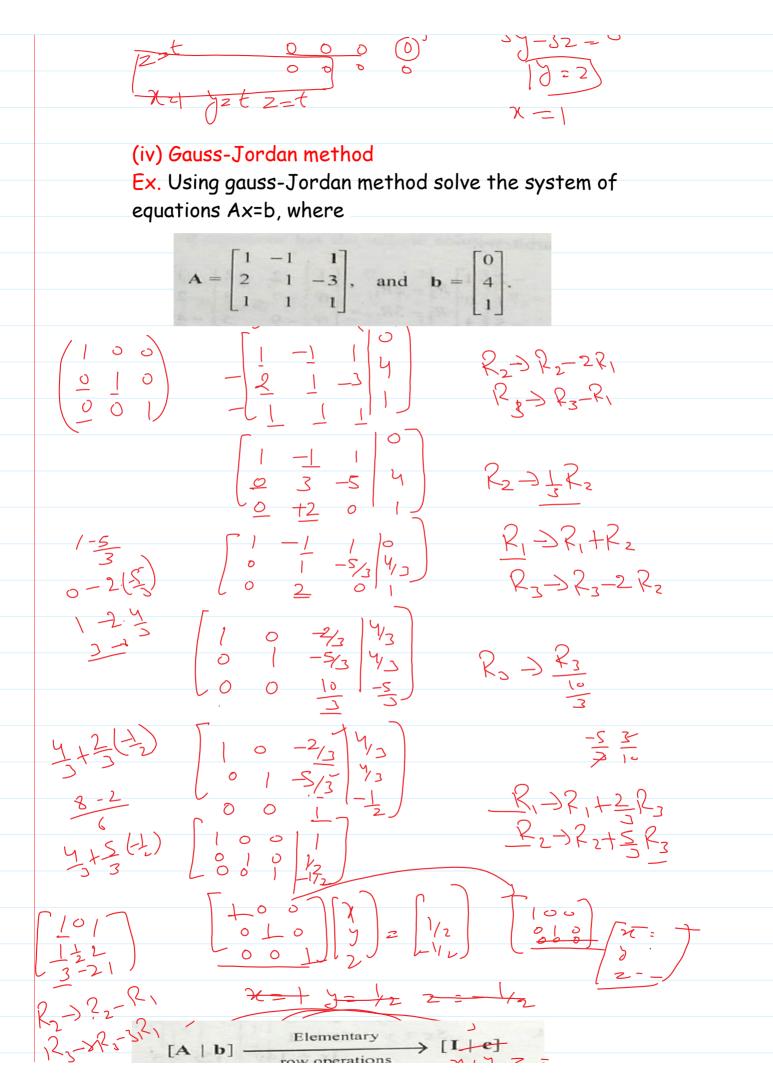
$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{bmatrix}.$$
Now we write the augmented matrix of order $m \times (n + 1)$

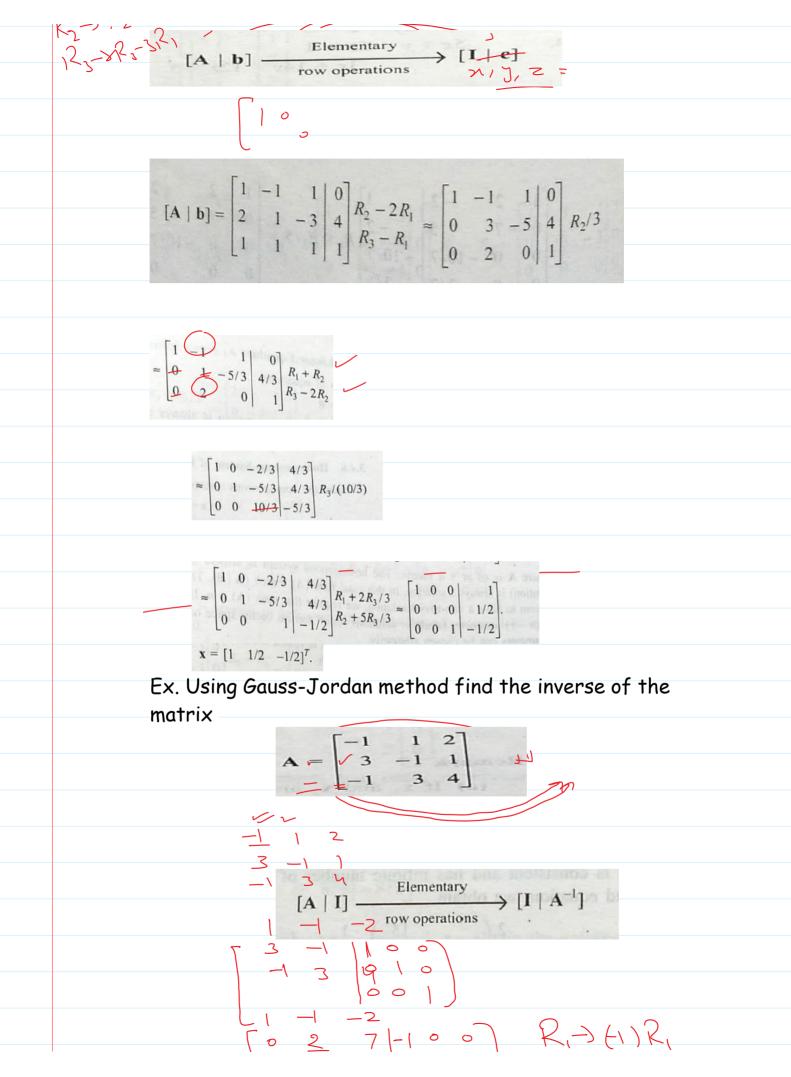
$$M_{1} \Rightarrow K_{1} \lor \qquad (\mathbf{A} + \mathbf{b}) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{1} \\ b_{2} \\ b_{2} \\ \vdots \\ b_{m} \end{bmatrix}$$
Now we can reduce this matrix in to row echelon form using elementary operations
$$M_{1} \Rightarrow K_{1} \lor \mathbf{b} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ 0 & \overline{a}_{22} & \cdots & \overline{a}_{2n} \\ \vdots \\ 0 & 0 & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{1} \\ \overline{b}_{2} \\ \vdots \\ b_{m} \end{bmatrix}$$

$$M_{2} \land M_{2} \land M$$

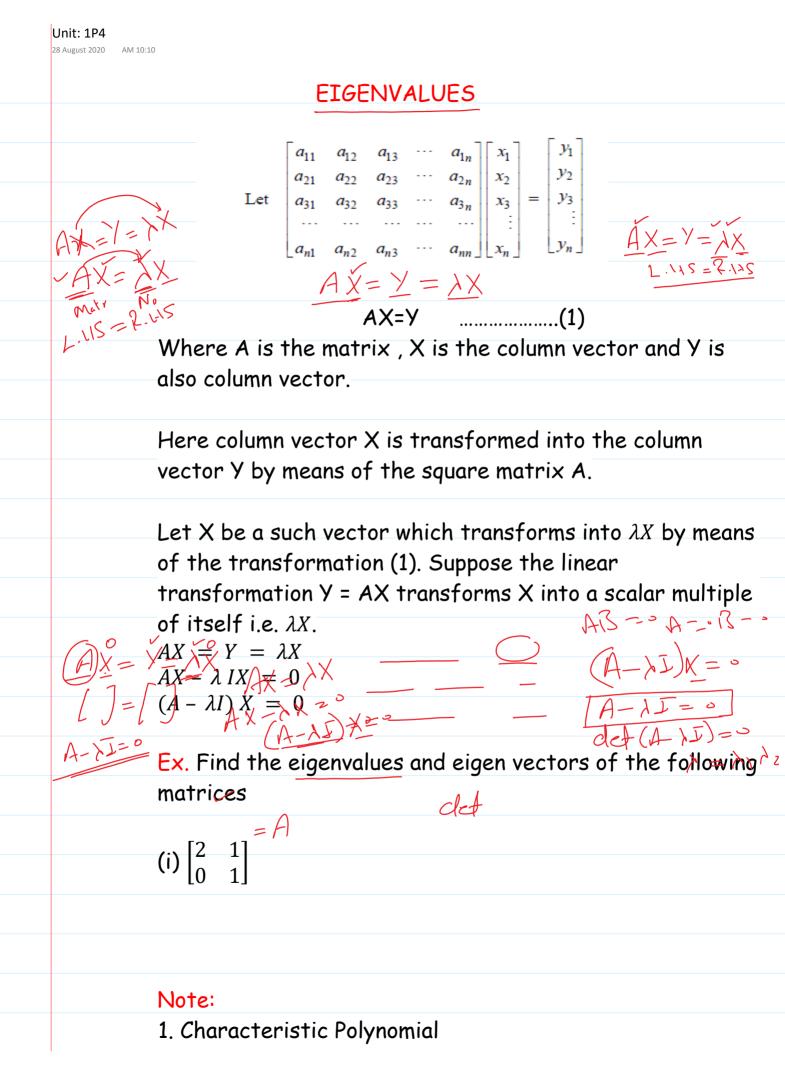
 $\chi - \gamma + 22 = -2$ (- $\chi + 2\gamma - 2 = 2$ _ 12) $\frac{R_2 \rightarrow R_2 - 2R_1}{R_3 \rightarrow R_3 + R_1}$ 2 1 -1 47 -1 2 -1 2 -) ~ - <u>1</u> ~ 2 z = -1 $\chi -1 - 2 = -2$ 3 + 5 = 8 R = 1L = 12 - 2 + 22 = -2>37-52 = 8 $\frac{2}{5}2 = -\frac{2}{2}$ (Ab) (Ab) (Ab) 1. Let $r < \underline{m}$ and one or more elements $b_{\underline{r+1}}^*, b_{\underline{r+2}}^*, \dots, b_{\underline{m}}^*$ are not zero. Then $rank(A) \neq rank(A|b)$ and the system of equation has no solution . 2. Let $m \ge n$ and r = n (the number of columns in A) and $b_{i+1}^*, b_{i+2}^*, \dots, b_{i}^*$ are all zero. In this case rank(A) = rank(A|b) = nP-n(A)=Penn(A/b) and the system of equations has unique solution. 3. Let r < n and $b_{r+1}^*, b_{r+2}^*, \dots, b_m^*$ are all zero. In this case x_1, x_2, \dots, x_n can be determined in term of remaining (n-r)







 $R_3 \rightarrow R_3 + R_1$ $\begin{array}{c} -1 & \circ \\ 3 & 1 & \circ \\ -1 & \circ \end{array} \end{array} \begin{array}{c} R_3 \rightarrow R_3 + R \\ R_2 \rightarrow \frac{1}{2} R_2 \end{array}$ $\begin{bmatrix} 1 & -2 & -1 & 0 & 0 \\ 0 & 1 & 7/2 & 7/2 & 7/2 & 0 \\ 0 & 2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ Rz-3R-271 $\begin{cases} 1 & 0 & \frac{1}{2} & \frac{1}$ 5-2153 $\begin{bmatrix} 10 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ AIF-



2. Characteristic Equation 3. Characteristic Roots or Eigenvalues
(ii) $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$
(iii) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$
$ \begin{array}{c} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{array} $
$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$
[2 1 1]
[4 0 0]
Note1: Direct Characteristic equation for matrix A
Order 2: $\lambda^2 - trac(A)\lambda + \det(A) = 0$ Order 3:
$\lambda^{3} - trac(A)\lambda^{2} + (Minor(a_{11}) + Minor(a_{22}) + Minor(a_{33}))\lambda - det(A) = 0$

Note2: The eigenvalue of

(a) a symmetric/Hermitian matrix are real

(b) a skew-symmetric/skew-Hermitian matrix are zero

or pure imaginary

(c)an orthogonal matrix are of magnitude 1 and are real or complex conjugate pairs

(d) an unitary matrix are of magnitude 1

Some Important Properties of Eigenvalues

(1) Any square matrix A and its transpose A' have the same eigenvalues.

(2) The sum of the eigenvalues of a matrix is equal to the trace of the matrix.

(3) The product of the eigenvalues of a matrix A is equal to the determinant of A.

(4) If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigen values of A, then the eigen values of

(i) kA are $k\lambda_1, k\lambda_2, ..., k\lambda_n$. (ii) A^m are $\lambda_1^m, \lambda_2^m, ..., \lambda_n^m$. (iii) A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, ..., \frac{1}{\lambda_n}$.

(5) $(A - kI)^{-1}$ has the eigenvalue $\frac{1}{\lambda - k}$.

(6) (A-kI) has the eigenvalue $\lambda - k$.

(7) For a real matrix A, if $\alpha + i\beta$ is an eigenvalue, then its conjugate $\alpha - i\beta$ is also an eigenvalue. When the Theorem: (Cayley-Hamilton Theorem) matrix A is complex this property does not hold its conjugate $\alpha - i\beta$ is also an eigenvalue. When the Theorem: (Cayley-Hamilton Theorem) matrix A is complex, this property does not hold. Every square matrix A satisfies its own characteristic equation

Ex. Verify Cayley-Hamilton theorem for the following matrices. Also find the inverse of the matrix.

$$\begin{array}{c} (i) \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \\ (ii) \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \\ (iii) \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

CHARACTERISTIC VECTORS OR EIGEN VECTORS

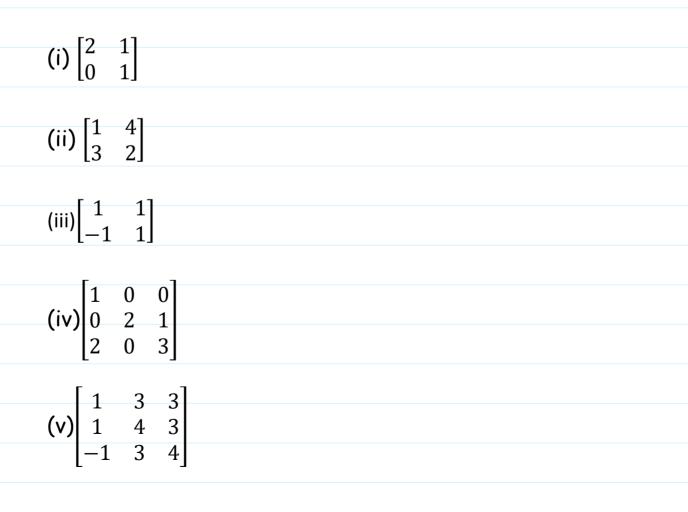
A column vector X is transformed into column vector Y by means of a square matrix A.

Now we want to multiply the column vector X by a scalar quantity λ so that we can find the same transformed column vector Y. i.e., $AX = \lambda X$ X is known as eigenvector. Show that the vector (1, 1, 2) is an eigen vector of the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$
 corresponding to the eigen value 2.

Note: Corresponding to each characteristic root λ , we have a corresponding non-zero vector X which satisfies the equation $[A - \lambda I] X = 0$. The non-zero vector X is called characteristic vector or Eigen vector.

Ex. Find the eigenvalues and eigenvectors of the following matrices



PROPERTIES OF EIGEN VECTORS:

1. The eigen vector X of a matrix A is not unique.

2. If $\lambda_1, \lambda_2, ..., \lambda_n$ be distinct eigen values of an n × n matrix then corresponding eigen vectors $X_1, X_2, ..., X_n$ form a linearly independent set.

3. If two or more eigenvalues are equal it may or may not be possible to get linearly independent eigenvectors corresponding to the equal roots.

4. Two eigenvectors X_1 and X_2 are called orthogonal vectors if $X'_1 X_2 = 0$.

5. Eigen vectors of a symmetric matrix corresponding to different eigenvalues are orthogonal.